Percolation in a random environment

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We consider bond percolation on the square lattice with perfectly correlated random probabilities. According to scaling considerations, mapping to a random walk problem and the results of Monte Carlo simulations the critical behavior of the system with varying degree of disorder is governed by new, random fixed points with anisotropic scaling properties. For weaker disorder both the magnetization and the anisotropy exponents are nonuniversal, whereas for strong enough disorder the system scales into an *infinite randomness fixed point* in which the critical exponents are exactly known.

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I. INTRODUCTION

Percolation is a paradigm for random processes [1], in which the ith bond (or site) of a regular lattice is occupied with a probability p_i which is generally taken independent of its position, $p_i=p$. In percolation theory one is interested in the properties of clusters, in particular in the vicinity of the percolation transition point $p=p_c$, when clusters with diverging size are formed. Using a close analogy with thermal phase transitions, which is based on the $Q\to 1$ limit of the ferromagnetic Q-state Potts model [2], a scaling theory has been developed and in two dimensions many, conjecturally exact results have been obtained by conformal field theory [3] and by Coulomb-gas methods [4].

In real systems, however, the occupation probabilities are generally inhomogeneous, i.e., position, direction or neighborhood dependent, and there are some correlations between them. The effect of quenched disorder, i.e., when the occupation probabilities are position dependent random variables, can be studied by scaling considerations. According to the Harris criterion [5] the relevance or irrelevance of the effect of quenched disorder on the percolation transition depends on the sign of the specific heat exponent α of the corresponding pure Potts model in the $Q\to 1$ limit. Since in any dimension $\alpha<0$ [1], the critical properties of ordinary and ''random'' percolation are equivalent. Another form of perturbations, e.g., long-range correlations between occupation probabilities [6] or anisotropy, such as in directed percolation [7], however, leads to modified critical properties.

In the present paper we consider the combined effect of disorder, anisotropy, and correlations, when the occupation probabilities are random variables, which are perfectly correlated in a d_d dimensional subspace. This type of behavior could be relevant to describe the properties of oil or gas inside porous rocks in oil reservoirs, when the rock has a layered structure [8]. For related earlier work see Ref. [9].

Models with perfectly correlated disorder play an important rôle in statistical physics and in the theory of (quantum) phase transitions. Among the early work we mention the partially exact solution of the McCoy-Wu model [10] (which is the two-dimensional Ising model with layered randomness) and the field-theoretical investigations by Boyanovsky and Cardy [11]. As a matter of fact in random quantum systems disorder is perfectly correlated along the (imaginary) time direction, i.e., here $d_d=1$. For these systems, in particular for random quantum spin chains, i.e., in $(1+1)$ dimension, many new, presumably exact results have been obtained recently by a strong disorder renormalization group (SDRG) method [12]. It was found that for strong enough initial disorder the critical behavior of several systems is governed by a so-called infinite randomness fixed point (IRFP) [13], with unusual scaling properties. Here we mention recent calculations on the random transverse-field Ising model (RTIM) [14,15], random quantum Potts and clock models [16], random antiferromagnetic Heisenberg spin chains [17,18], and ladders [19] and also nonequilibrium phase transitions in the presence of quenched disorder [20]. In many cases a crossover between weak and strong disorder regimes has been observed and a general scaling scenario has been proposed [21].

In the present paper, we study percolation on a square lattice with strip random occupation probabilities. We investigate the critical behavior of the system with varying strength of disorder by scaling considerations, by random walk mappings and by Monte Carlo (MC) simulations. The structure of the paper is the following. The model and the relevant physical quantities are introduced in Sec. II. Investigations in the weak and strong disorder limits are given in Sec. III, MC simulations for intermediate disorder are presented in Sec. IV. The paper is concluded by a discussion in Sec. V.

II. THE MODEL

We consider bond percolation on a square lattice with sites $\{i,j\}$, $1\leqslant i\leqslant L$ and $1\leqslant j\leqslant K$, where the occupation probabilities, $0<p<1$, are random variables, which are perfectly correlated (identical) along vertical lines as indicated in Fig. 1. If the average value of the occupation probabilities exceeds a critical value $\langle p\rangle>p_c$ there is a percolation transition in the system. The value of p_c can be determined by noticing that under a duality transformation, which maps the ordered and the disordered phases of the system into each other, the layered structure of the system is preserved and the

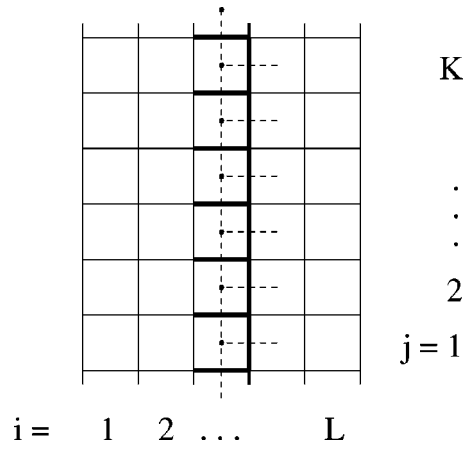

FIG. 1. Percolation on the square lattice with random bond occupation probabilities, which are perfectly correlated along the vertical direction. The portion of the lattice having the same occupation probability p is denoted by bold lines, whereas the corresponding part of the dual lattice with $\tilde{p}=1-p$ is shown by dashed lines.

dual value of the local probability is transformed as: $\tilde{p}=1-p$ [22]. Consequently the probability distribution, $P(p)$, is transformed into $\tilde{P}(\tilde{p})=P(1-p)$ and the random system is self-dual, if the probability distribution is symmetric: $P(p)=P(1-p)$ and thus the average value of p is given by $\langle p\rangle = p_c = 1/2$. Since there is one phase transition in the system, the self-duality point corresponds to the critical point and the distance of the critical point, t is defined as

$$t=\langle p\rangle - p_c . \tag{1}$$

In the presence of quenched disorder the mean value of a physical observable Φ is calculated as $[\langle\Phi\rangle]_{\rm av}$, where $\langle\cdots\rangle$ denotes thermal averaging for a given realization of the disorder and $[\cdots]_{\rm av}$ stands for disorder averaging. In percolation the basic quantities of interest are the fractal and connectivity properties of the largest clusters. In the following we use the concept of anisotropic scaling [23], when the correlation lengths in the two directions, (which correspond to the extensions of the largest clusters) involve different critical exponents: $\xi_\perp \sim |t|^{-\nu_\perp}$ and $\xi_\parallel \sim t^{-\nu_\parallel}$. Thus the anisotropy exponent

$$z=\frac{\nu_\parallel}{\nu_\perp} \tag{2}$$

is generally different from one. In the ordered phase, $t>0$, the number of points belonging to the infinite cluster N_0 scales around the transition point as

$$N_0 = L K t^{\beta} \tilde{N}(L t^{\nu_\perp}, K t^{\nu_\parallel}), \tag{3}$$

where β is the critical exponent of the order parameter. At the critical point, $t=0$, fixing the ratio $K/L^z = O(1)$ we obtain

$$N_0 \sim L^{d_\perp} \sim K^{d_\parallel}, \tag{4}$$

where the two fractal dimensions of the infinite cluster are given by

$$d_\perp = 1 + z - \beta/\nu_\perp \tag{5}$$

and $d_\parallel = d_\perp / z$. The distribution of cluster sizes $R(N)$ at the critical point asymptotically behaves as

$$R(N)\mathrm{d}N = N^{-\tau}\tilde{R}(N/L^{d_\perp})\mathrm{d}N, \tag{6}$$

where $\tau = 2 + \beta/(\nu_\perp d_\perp)$. This relation can be obtained by generalizing the similar result for ordinary percolation [1].

Correlation between two sites with coordinates, $\{i_1, j_1\}$ and $\{i_2, j_2\}$, is defined as the expectation value of the connectivity, $\delta(\{i_1,j_1\},\{i_2,j_2\})$, which is 1, if the two sites belong to the same cluster and zero otherwise. Here we mainly consider correlations in the perpendicular direction

$$C_\perp(i_1,i_2) = \frac{1}{K}\sum_{j=1}^{K}\left[\langle \delta(\{i_1,j\},\{i_2,j\})\rangle\right]_{\rm av}, \tag{7}$$

where an average over the vertical coordinate, $j_1 = j_2 = j$ is also performed. When correlations in the bulk are calculated we use periodic boundary conditions (BC), (thus $i = L+1 \equiv 1$), take maximal distance between the sites, $i_2 = i_1 + L/2$, and average over the position i_1. The average bulk correlations, calculated in this way, scale at the critical point as

$$C_\perp^b(L) \sim L^{-\eta_\perp}, \tag{8}$$

where $\eta_\perp = 2\beta/\nu_\perp$. We also considered the system with free boundaries at $i=1$ and $i=L$ and calculated the correlations between two surface sites. This end-to-end correlation function at the critical point asymptotically behaves as

$$C_\perp(1,L) \equiv C_\perp^s(L) \sim L^{-\eta_\perp^s}, \tag{9}$$

where the decay exponent $\eta_\perp^s$ is related to the surface fractal properties of the infinite cluster. Closing this section we quote the values of the critical exponents for two-dimensional ordinary percolation [24]

$$\nu^{(0)} = 4/3, \quad \eta^{(0)} = 5/24, \quad \eta^{s(0)} = 2/3. \tag{10}$$

III. STRENGTH OF DISORDER: LIMITING CASES

The strength of disorder Δ is related to the broadness of the probability distribution, $P(p)$. In terms of the integrated probability distribution, $\Pi(p) = \int_0^p P(p')\mathrm{d}p'$, we introduce the probabilities, $p_{1/4}$ and $p_{3/4}$ with the definitions: $\Pi(p_{1/4}) = 1/4$ and $\Pi(p_{3/4}) = 3/4$. Since the central half of the distribution is located in the region: $p_{1/4} \leqslant p \leqslant p_{3/4}$ its relative width is measured by

$$\Delta = \frac{p_{3/4} - p_{1/4}}{1 - p_{3/4} + p_{1/4}}, \tag{11}$$

what we can identify with the strength of disorder.

In this paper, we used two specific forms of the distribution. For the bimodal distribution ($0 \leqslant q \leqslant 1/2, \bar{q} = 1-q$):

$$P_{bin}(p) = \frac{1-t}{2}\delta(p-q) + \frac{1+t}{2}\delta(p-\bar{q}), \qquad (12)$$

the critical point is located at $t=0$ and the strength of disorder is given by $\Delta_{bin} = (1-2q)/2q$. Thus, as expected the bimodal disorder is weak for $q \approx 1/2$ and strong for $q \ll 1/2$.

The other distribution we use has a power-law form

$$P_{pow}(p) = \frac{1}{D2\bar{p}}\left(\frac{p}{\bar{p}}\right)^{-1+1/D} \qquad 0 < p < \bar{p} < 1, \qquad (13)$$

and $P_{pow}(1-p) = P_{pow}(p)\bar{p}/(1-\bar{p})$, for $\bar{p} < p < 1$. The distance from the critical point is measured by $t = (\bar{p} - 1/2)/(D+1)$, and for $\bar{p} = 1/2$, i.e., for $t=0$ the distribution is indeed symmetric. In this case the strength of disorder is given by $\Delta_{pow} = 2^D - 1$. Thus for $D=0$ we recover the ordinary percolation and the strength of disorder is monotonically increasing with D. Therefore, D will be often called as the disorder parameter of the distribution.

A. Weak disorder

In the limit of weak disorder one usually decides about the relevance-irrelevance of the perturbation by performing a stability analysis at the ordinary percolation fixed point. Generalizing the method by Harris [5] the crossover exponent due to correlated disorder is calculated as

$$\phi = 2 - \nu^{(0)} = 2/3, \qquad (14)$$

where we used $\nu^{(0)} = 4/3$ in Eq. (10). Since $\phi > 0$, even weak correlated disorder is a relevant perturbation, thus a new random fixed point is expected to control the critical behavior of the model.

B. Strong disorder: Mapping to random walks

Next we turn to study the behavior of the system for extremely strong disorder using the bimodal distribution in Eq. (12) in the limit $q \to 0$. In this limiting case the percolation in a given layer with a probability p_i has a simple, anisotropic structure (for an illustration see Fig. 2). If this probability is extremely large, $p_i = \bar{q}$, then here almost all bonds are occupied, except of a very small fraction of q. Since the typical distance between two nonoccupied bonds is $l \sim 1/q$, the cluster in the ith layer is composed of long connected units of typical size l. On the other hand, if the probability is extremely small, $p_j = q$, then almost all bonds in this layer are unoccupied, except of a very small fraction of q. Since the typical distance between two occupied bonds is $l \sim 1/q$, the cluster in the jth column is composed from long empty units of typical size l. Notice the duality in the structure of the two types of column.

With this prerequisite we consider the order parameter in the surface column, $m_s(L)$, which is the fraction of surface sites belonging to a cluster of horizontal extent L. In order to make a statement about the value of $m_s(L)$ we consider parallel strips of width $k \leqslant L$ and introduce the quantity, n_k, as the typical number of bonds at the kth (i.e., surface) column of a cluster, which is connected to the other surface of the strip. Starting with $k=1$ we have two possibilities. For extremely small probability $p_1 = q$ there is no surface cluster in the system, thus we have $n_1 = 0$. Otherwise, for $p_1 = \bar{q}$, a surface site is connected to all sites of a "connected unit" of length l, thus we have $n_1 \sim l \sim 1/q$. For $k=2$, if the probability is extremely large in the second layer, too, $p_1 = p_2 = \bar{q}$, then a surface cluster extends up to the second layer and its vertical size, which is given by n_2, can be estimated as follows (see Fig. 2). The end of a cluster is signalled by the fact that in both columns unoccupied bonds are in neighboring positions, which happens with a probability q^2, from which the typical size of a cluster $n_2 \sim 1/q^2 \sim n_1/q$, follows. Repeating this argument for $p_i = \bar{q}$, $i = 1,2,\ldots,k$ we obtain $n_k \sim 1/q^k \sim n_{k-1}/q$. Now having a small probability at the following layer, $p_i = \bar{q}$, $i = 1,2,\ldots,k$ and $p_{k+1} = q$, then only a fraction of q of the sites n_k have a further connection, thus n_k will be reduced by a factor q giving $n_{k+1} \sim n_k q$. Inclusion of any further layer with an extremely small probability will reduce n_j by a factor of q, until we arrive at $n_{j'} = O(1)$, when for the next small probability layer we have $n_{j'+1} = 0$, thus the surface cluster ends at this distance.

From this example we can read that the n_k numbers are either integer powers of $1/q$, $n_k \sim 1/q^{X_k}$, for $X_k = 0,1,\ldots$, or $n_k = 0$, if formally $X_k < 0$. Furthermore, we have the transformation rules

$$n_{k+1} \sim \begin{cases} n_k/q, & p_{k+1} = \bar{q} \\ n_k q, & p_{k+1} = q, \end{cases} \qquad (15)$$

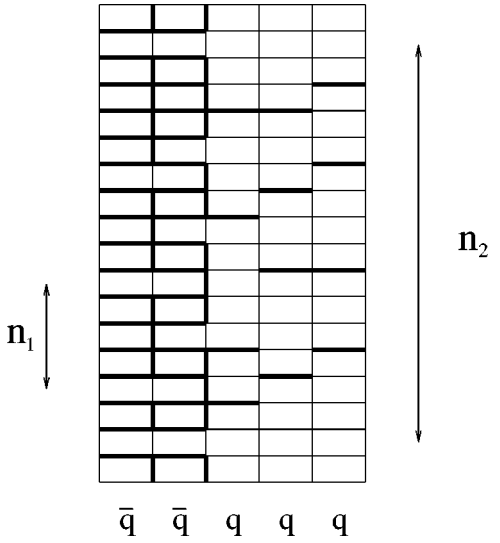

FIG. 2. Structure of the percolation cluster in the extreme bimodal distribution with $p_1 = p_2 = \bar{q}$ and $p_3 = p_4 = p_5 = q$ (here with $q \approx 1/4$). In a layer with extremely large (small) probability there are connected (empty) units of typical length $l \sim 1/q$. The number of sites of the connected cluster at the other surface of a strip of width, k, n_k is given by $n_1 \sim 1/q$, $n_2 \sim 1/q^2$, $n_3 \sim 1/q$, and $n_4 = O(1)$ (see text). In the limit $q \to 0$ the cluster ends at $k=4$, thus $n_5 = 0$.

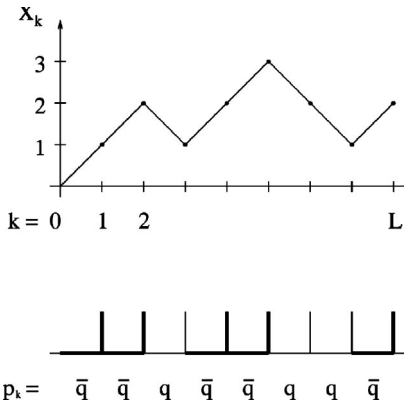

FIG. 3. Illustration of the RW mapping of percolation for a given realization of the extreme binary distribution. Layers with high, $\overline{q}$, (low, q) probabilities are drawn by thick (thin) lines and the corresponding RW makes a step of unit length upwards (downwards). The position of the RW, in the kth step X_k is related to, n_k, the number of typical sites in the kth layer of percolation, which are connected to a given surface site as $n_k \sim q^{-X_k}$. The surface cluster extends to a distance L if $X_k \geqslant 0$, for all $k=1,2,\ldots,L$, thus the RW has a surviving character.

where in the second case $n_{k+1}=0$, if $n_k = O(1)$. At this point we can formulate the condition that the surface order parameter in a given sample (in a rare realization) is $m_s(L) = O(1)$, if $n_k \geqslant O(1)$, for all $k=1,2,\ldots,L$. For all other cases $m_s(L)=0$. Consequently to calculate the *average value* of $m_s(L)$ it is enough to find the fraction of rare realizations ρ_L^s for which $m_s(L)=O(1)$, since $[m_s]_{\rm av} \sim \rho_L^s$. To calculate ρ_L^s we use a random walk (RW) mapping (see an illustration in Fig. 3), in which to each disorder realization we assign a one-dimensional RW, which starts at $X_0=0$ and takes its kth step upwards, $x_k=1$ (downwards, $x_k=-1$) if the corresponding bond occupation probability is extremely large, $p_k=\overline{q}$ (extremely small, $p_k=q$). The position of the walker at the kth step, $X_k=\Sigma_{i=1}^k x_k$ is related to n_k as $n_k \approx q^{-X_k}$. Then, as argued before, the surface cluster extends upto a vertical distance L if $X_k \geqslant 0$, for every $k=1,2,\ldots L$, i.e., the RW has a surviving character.

At the critical point of the percolation problem, $t=0$, the corresponding RW is unbiased, and the fraction of surviving L-step RWs scales as $\rho_L^s \sim L^{-1/2}$. Now the fraction of clusters which connect the two free boundaries of the strip over a distance L, and thus contribute to the average end-to-end correlations in Eq. (9), is given by $(\rho_{L/2}^s)^2$, since at each site there should be an independent percolating surface cluster, which meet in the middle of the system. Consequently the average end-to-end correlations at the critical point scale as $C_\perp^s(L) \sim L^{-1}$ thus the corresponding decay exponent in the strong disorder limit is given by

$$\eta_\perp^{s,(\infty)} = 1. \tag{16}$$

Using the RW mapping one can easily estimate the perpendicular size of the percolating clusters, which is given by $\xi_\parallel(L) \sim n_{L/2} \sim q^{X_{L/2}}$. Since the transverse fluctuations of unbiased surviving RWs scale as $X_{L/2} \sim L^{1/2}$ we obtain in the strong disorder limit

$$\ln \xi_\parallel \sim \xi_\perp^{1/2}. \tag{17}$$

Consequently the anisotropy exponent z in Eq. (2) is formally infinite for strong disorder.

Other results can be simply obtained by noticing that the same type of RW mapping applies to the one-dimensional RTIM [25], too, so that we can simply borrow the results obtained in this case.

For bulk correlations one should consider the fraction of realizations ρ_L for which a given bulk site belongs to a connected cluster of vertical size L. As was shown in Ref. [26] for these realizations the *thermal average* of the position of the RW has a surviving character. The fraction of these walks is given by [27,26]: $\rho_L \sim L^{-(3-\sqrt{5})/4}$, consequently the critical average bulk correlations being $C_\perp^b(L) \sim (\rho_L)^2$ have a decay exponent

$$\eta_\perp^{(\infty)} = \frac{3-\sqrt{5}}{2}, \tag{18}$$

in the strong disorder limit.

Finally, outside the critical point the mapping is related to a biased RW, with a finite drift velocity, which is proportional to t. From the surviving probability of biased RWs one obtains for the correlation length critical exponent [25]

$$\nu_\perp^{(\infty)} = 2. \tag{19}$$

The scaling exponents and relations in Eqs. (16)–(19) are identical with those of the IRFP of the one-dimensional RTIM [14], which is known to control the critical behavior of several other random quantum spin chains [16,21] and nonequilibrium phase transitions in the presence of quenched disorder [20]. At this point our next question is about the region of attraction of the IRFP. For the RTIM, where the RW mapping can be generalized for weaker disorder, any small amount of randomness seems to bring the system into the IRFP [14], which claim is checked by intensive numerical calculations [28,25,26]. There are, however, several other models [random quantum clock-model, Ashkin-Teller model [21], directed percolation [20], $S=1$ random antiferromagnetic spin chains [18], etc.] where weak disorder is not sufficient to bring the system into the IRFP. In these cases either the pure systems fixed point stays stable against weak disorder perturbations or the competition between (quantum) fluctuations and weak quenched disorder leads to conventional random scaling behavior. For the random percolation problem the latter scenario is likely to happen, since the RW mapping cannot be extended for small disorder. (The transformation law for the connected sites, $n_k/n_{k-1} \approx q$ or $1/q$, does not hold around $q \approx 1/2$.) We are going to study this issue numerically by MC simulations in the following section.

IV. NUMERICAL RESULTS

For intermediate strength of disorder we studied the percolation by MC simulations. Since the critical properties of the problem are related to the connectivity properties of clus-

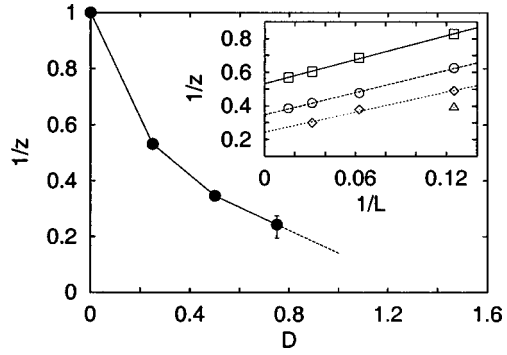

FIG. 4. Estimates for the anisotropy exponent for different strength of disorder. The straight lines connecting the points are guides to the eye, for $D>D_\infty\approx 1.2-1.5$ the anisotropy exponent is possibly divergent. In the inset extrapolation of the size-dependent effective anisotropy exponents is shown for $D=0.25$ 0.5 0.75, and 1.0, up to down.

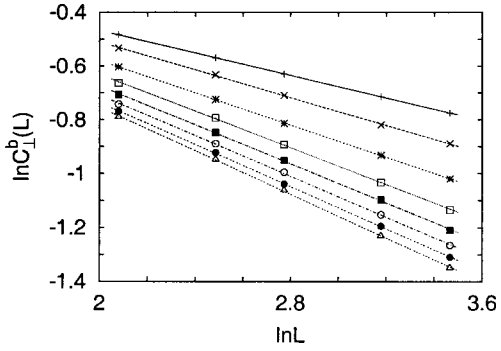

FIG. 5. Average bulk correlations vs the width of the strip for different strengths of disorder, from $D=0$ to $D=1.75$ in units of 0.25 from up to down. The typical error is generally smaller than the size of the symbols for small D, whereas for larger D it is at most twice of the size of symbols. The straight lines are least-square fits.

ters for this purpose we implemented the standard Hoshen-Kopelman labeling algorithm [29]. To decide about the shape of the lattice one should take into account the expected anisotropic scaling properties of the system, since the scaling functions, as in Eq. (3) depend on the ratio $r=L^z/K$, where z is an unknown parameter. To overcome these difficulties we used a striplike geometry, when $K\gg L$, thus $r\approx 0$ for all strip widths. In practice we had $K=10^5$, went upto $L=64$ and imposed periodic BC in the vertical direction. For the distribution of the disorder we used the power-law form in Eq. (13), which has turned out to be successful in similar investigations for random quantum spin chains [21]. Since averaging in the vertical direction in Eq. (7) (and also in the horizontal direction for bulk correlations) is equivalent to a partial average over quenched disorder it was enough to consider only a limited number ($\sim 10-20$) realizations.

First, we determine the anisotropy exponent z by calculating the probability distribution of clusters in Eq. (6). While the decay exponent τ in Eq. (6) has only a weak anisotropy dependence, the scaling function $\tilde{R}(y)$ turned out to be sensitive of the value of z. As we noticed in the numerical calculations $\tilde{R}(y)$ has two different regimes. For smaller values of the parameter, $y=N/L^z<y^*$, the finite size effects are negligible and the scaling function is approximately constant. For $y>y^*$, when the largest clusters touch the boundaries, the scaling function has a characteristic variation. Measuring the position of y^* for different widths L we obtained a series of effective anisotropy exponents, which are then extrapolated to $L\to\infty$, as shown in the inset to Fig. 4. This procedure is repeated for several disorder parameters and the extrapolated anisotropy exponents are plotted in Fig. 4. Unfortunately, with this method we could not go to very strong disorder, while the crossover region cannot be clearly located for $D>1$. However, it is clear from the available data that z is monotonically increasing with the strength of disorder and it is likely that z will be divergent for $D>D_\infty\approx 1.2-1.5$.

In order to obtain more information about the critical behavior of the system we have calculated the bulk and the end-to-end average correlation functions at the critical point, as defined in Eqs. (8) and (9), respectively. In Fig. 5 the average bulk correlations $C_\perp^b(L)$ vs L is drawn in a log-log plot. The slope of the curves, which is related to the decay exponent $\eta_\perp$ has a disorder dependence.

The exponents, calculated in this way together with the decay exponent of the end-to-end correlations $\eta_\perp^s$ are plotted in Fig. 6. As seen in Fig. 6 both exponents are monotonously increasing with the strength of disorder and tend to saturate at the respective IRFP values, given in Eqs. (18) and (16). The value of disorder strength, where the saturation takes place, within the error of the calculation, is the same for the two exponents and it is compatible with the estimate D_∞ as calculated from the divergence of the dynamical exponent in Fig. 4.

We can thus conclude that the critical behavior of the random percolation process has a weak-to-strong disorder crossover. For weaker disorder, $D<D_\infty$, what we call the *intermediate disorder regime*, the critical behavior of the system is controlled by a line of conventional fixed points. Here the anisotropy exponent is finite, and together with the order-parameter exponents, $\eta_\perp$ and $\eta_\perp^s$, monotonously increasing with the strength of disorder. In the *strong disorder regime*, $D>D_\infty$, the critical behavior of the system is controlled by the IRFP. Here the anisotropy exponent is formally infinity

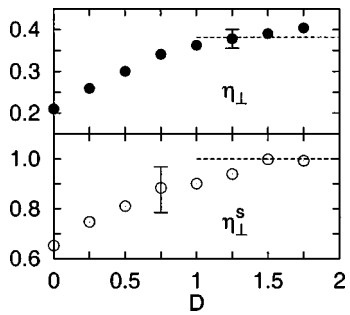

FIG. 6. Bulk ($\eta_\perp$) and surface ($\eta_\perp^s$) decay exponents vs the strength of disorder. Values at the IRFP, as given in Eqs. (18) and (16), are denoted by dashed lines. Two typical error bars are also indicated.

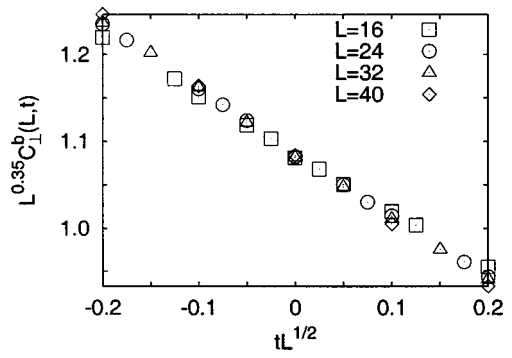

FIG. 7. Scaling plot of the bulk correlation function with $\nu_\perp = 2$ at a disorder strength $D = 0.75$.

and the other critical exponents have no disorder dependence.

The nonuniversal nature of the critical behavior in the intermediate disorder regime is possibly connected to the presence of a marginal operator, which should have vanishing anomalous dimension, $x_e = 0$, in the entire disorder range, $0 < D < D_\infty$. In our case the disorder perturbation is connected to the local energy-density operator, for which the marginality condition, according to the Harris criterion in Eq. (14) requires the condition $\phi = 0$, thus $\nu_\perp = 2$. To verify this scenario we have calculated the average bulk correlation function $C_\perp^b(L,t)$ outside the critical point, at a disorder strength $D = 0.75$ which is in the middle of the intermediate disorder regime. According to scaling considerations

$$C_\perp^b(L,t) = L^{-\eta_\perp} \tilde{C}(tL^{1/\nu_\perp}), \tag{20}$$

thus from an optimal scaling collapse $\nu_\perp$ can be determined. As shown in Fig. 7 the scaling behavior of $C_\perp^b(L,t)$ is compatible with the conjectured value of $\nu_\perp = 2$ and thus with the marginality condition.

V. DISCUSSION

In this paper, bond percolation is studied on the square lattice with strictly correlated, layered randomness. The phase diagram of the problem as a function of the strength of disorder contains two regions. For strong enough disorder, the critical properties of the model are controlled by an IRFP, the properties of which are exactly known by a RW mapping. For weaker disorder, in the intermediate disorder regime the critical behavior is found to be controlled by a line of conventional random fixed points, where both the anisotropy exponent and the order-parameter exponents are disorder dependent. The correlation length exponent, however, stays constant at its marginal value.

This type of critical behavior is very similar to that obtained in a class of random quantum spin chains [21,20]. This close similarity can be understood by noting the relation between percolation and the $Q \to 1$ limit of the Q-state ferromagnetic Potts model. With layered randomness the two-dimensional Potts model in the Hamiltonian limit [30] is equivalent to a quantum Potts chain, with random couplings J_i and transverse fields h_i [31], the critical behavior of which can be studied by the SDRG method [16]. In this procedure the couplings and transverse fields are put in descending order and the strongest terms are successively decimated out, whereas neighboring terms are replaced by renormalized values. Decimating the strongest coupling, say J_2, yields a new effective spin cluster in a renormalized transverse field of strength

$$\tilde{h} = \frac{2}{Q} \frac{h_1 h_2}{J_2}, \tag{21}$$

where h_1 and h_2 are the original transverse fields acting at the two end-spins of J_2. Similarly, if, the spin in the strongest transverse field h_2 is decimated out, then a new renormalized coupling is generated between remaining spins, which is of the form in Eq. (21), by interchanging $h_i \leftrightarrow J_i$, which is due to duality.

If the disorder is strong enough, so that the system under renormalization is in the attractive region of the IRFP, the model specific prefactor $2/Q$ in Eq. (21) does not matter and the critical properties are universal. The region of strong attraction of the IRFP, however, is limited by $Q = 2$, i.e., for the RTIM. For smaller values of Q, like in percolation, when the prefactor in Eq. (21) is larger than one, for weak disorder some renormalized couplings and transverse fields are larger than the decimated ones. If this happens frequently, i.e., when the disorder is too weak, then the SDRG method is no longer valid and the critical behavior of the model is expected to be controlled by a conventional random fixed point. This is exactly what we obtained by MC simulations.

We conclude our paper with two remarks. First, for strong enough disorder the critical behavior of both ordinary and directed percolation [20] is controlled by the same IRFP, thus the original anisotropy between the two pure problems does not make any influence about the (strongly) random critical behavior. Our second remark concerns possible Griffiths effects in the random percolation problem. Using the analogy with random quantum spin chains for strong disorder some dynamical quantities of the random percolation problem are singular also outside the critical point. For example the susceptibility in a uniform field, H diverges as $\chi \sim H^{-1+1/z'}$, and the vertical correlation function decays algebraically as $C_\parallel(l) \sim l^{-1/z'}$, where z' is a finite dynamical exponent, which depends on the distance of the critical point.

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